

ON THE ANALYSIS OF CLEAR AIR TURBULENCE BY USE OF RAWINSONDE DATA¹

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ABSTRACT

Regions of clear air turbulence in the upper troposphere and lower stratosphere are classified into four groups, and the characteristics of each group are summarized. An empirical turbulence index is defined that describes meteorological conditions associated with a type of turbulent region that is relatively large and that sometimes contains severe turbulence. The turbulence index, Richardson's number, and other meteorological quantities are described in relation to a clearly defined case of turbulence observed by a research aircraft. For the period March 12-24, 1962, vertical shear, lapse rate, Richardson's number, and the turbulence index were calculated by electronic computer from rawinsonde data over the United States. These computed quantities are compared with pilot reports of turbulence. Individual maps are presented that illustrate substantial agreement between computed quantities and turbulence reports. Standard statistical tests show that both Richardson's number and the turbulence index have definite skill in turbulence analysis. Suggestions are given for further improvement of criteria for analyzing turbulence. To enhance turbulence research and operational analysis, the writers believe that a turbulence sensor operated as an integral part of the radiosonde system is very desirable.

1. INTRODUCTION

The meteorological characteristics of regions of clear air turbulence have been described in several recent papers [1, 2, 3, 4, 6, 11, 13]. On the basis of wind and temperature fields, particularly on the mesoscale, it appears that turbulent regions may be classified as follows:

A. One type of region in which turbulence is often observed consists of quasi-horizontal layers of appreciable wind speed in which wind direction changes markedly in the vertical and horizontal. These conditions of flow are especially favorable to turbulence when they occur at the boundary of an upper front or at the tropopause. Layers with these characteristics are usually located within a few hundred miles laterally from a jet stream, and occur more frequently in and downstream from sharp troughs and ridges than in straight or gradually curved portions of the flow. In sharp troughs, such turbulent regions are generally at or slightly below the level of the jet core, while in ridges they are above the core. The vertical wind shear vector ($\partial \mathbf{V} / \partial z$) is large due to the directional change. A large vertical change in wind speed may or may not exist in the locale of this type of turbulence. In either case, the large shear vector tends to make the Richardson number small. Horizontal shear is usually cyclonic. Mesoscale vertical motions having intensities on the order of a meter per second are found in and near the turbulent regions. The turbulent areas may be 100 mi. wide and some hundreds of miles long parallel to the flow direction. The intensity of the turbulence may be severe. These meteorological conditions are consistent

with Kuettner's [8] hypothesis that unstable gravity-inertia waves produce the eddies (75 to 800-ft. wavelengths) that are turbulent to a subsonic jet aircraft. Regions of this type are frequently detectable by analysis of upper-air data of the density that exists over the United States.

B. A second type of turbulent region consists of areas near pronounced jet cores (within approximately 75 mi. on the cyclonic side of the core and within approximately 3000 ft. in the vertical). In these regions, vertical shear changes from strongly positive below the core to strongly negative above; i.e., $\partial^2 V / \partial z^2$ has a large negative value [12]. (Dynamically, the change in shear represents a stabilizing effect that tends to prevent vertical penetrations through the jet core and maximum wind level.) The lapse rate is generally moderately unstable. Pronounced turning of wind with height is not observed in the vertical; however, there are indications from aircraft data that strong horizontal convergence (or divergence) exists on the mesoscale as a result of lateral changes of wind direction and speed. The turbulent areas are generally smaller than those of type A described above. Turbulence intensity may be moderate or severe in some cases. These regions are difficult to detect from upper-air data because of their relatively small size and because of the loss of some wind reports in well developed jet streams.

C. A third type of turbulent area occurs in anticyclonic horizontal shear (generally south of jet streams) below the maximum wind level. Vertical wind shear and turning of the wind with height do not have unusual magnitudes; however, moderately unstable lapse rates tend to

¹ This research was performed under U.S. Weather Bureau Contract No. Cwb-10624.

give smaller than average values of Richardson's number. Dynamic instability $[(\zeta+f)<0]$ may be present in the turbulent region. Also, cirrus clouds may be present in the same general area but their presence is not believed to be of importance in affecting the turbulent intensity. The turbulent areas may be quite large but the intensity is generally light or light-to-moderate.

D. It is well known that mountain waves contain highly turbulent conditions in rotors and sometimes at the tropopause [9]. Thus, mountain-wave turbulence may be considered as another type of free air turbulence. Even when well developed mountain waves are not present, mountainous terrain appears to increase somewhat the size and intensity of turbulent regions in comparison with turbulence expected with the same flow conditions elsewhere. Similarly, increased probabilities of turbulence apparently exist downstream from areas of deep convection.

Synoptic-scale features of the upper flow in turbulent regions (determined from large numbers of commercial and military pilot reports) have been investigated by Colson [1,2,3]. These studies show that turbulent areas are sometimes very large (perhaps 1000 mi. along the flow direction) and may persist for periods up to a day (as on March 14 and 15, 1962, south of the Great Lakes). In Colson's February 1963 data, turbulent areas showed definite relationships to mean trough and ridge positions.

Taken together, the synoptic-scale and mesoscale characteristics suggest a relationship as follows. In certain portions of the synoptic-scale flow patterns, inertial forces tend to produce large mesoscale variations of wind, vertical motions, and temperature. These mesoscale features provide conditions favorable for the initiation and growth of unstable wave motions that degenerate into turbulent eddies. The mesoscale "generating" regions of turbulence tend to travel with synoptic-scale patterns (troughs, ridges, and jet streams). As wind speeds in the upper flow are generally faster than pattern motions, air parcels may overtake and pass through the generating regions. Theoretically, the continuation or cessation of turbulence, once initiated, depends on the local Richardson number.

The main purpose of this paper is to investigate objective methods of identifying turbulent regions from standard upper-air data, assuming that the descriptions of turbulent regions given above are essentially correct. In section 2 a "turbulence index" intended to identify turbulent regions of type A is defined, and in section 3 its application is illustrated in a case of moderate turbulence observed by a research aircraft. In sections 4 and 5, the utility of the turbulence index and Richardson's number is tested in individual cases and statistically by comparison with turbulence reports for the period March 12 to 24, 1962.

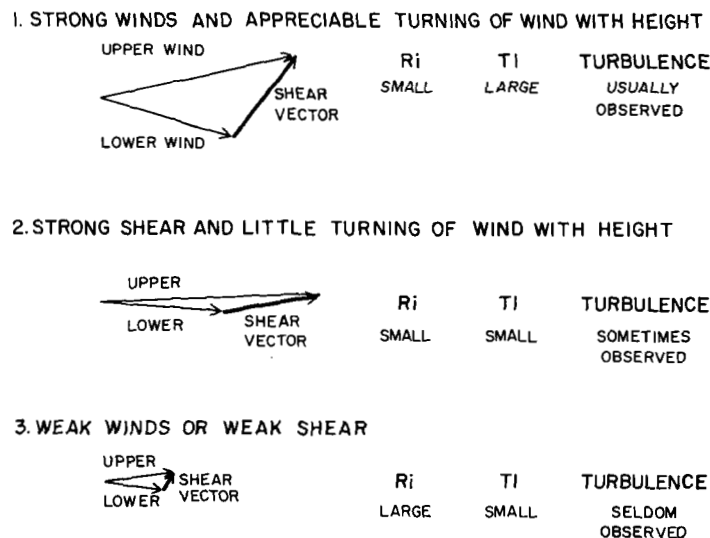


FIGURE 1.—Comparison of Richardson's number and the turbulence index under typical conditions.

2. DISCUSSION OF TURBULENCE CRITERIA

In order to identify turbulent regions of type A, the product of wind speed (V) and change of wind direction (α) with height was computed for several cases of moderate or severe turbulence [4]. It was found that patterns of $|V(\partial\alpha/\partial z)|$ related well to turbulent regions of type A in several cross-sectional flights through jet streams (large magnitudes were associated with turbulence and small magnitudes with smooth flight). The relationship of turbulence to a vertical change in wind speed ($\partial V/\partial z$) appeared to be weaker than with $V(\partial\alpha/\partial z)$. (This empirical result should be checked by means of further detailed measurements of winds and turbulence.) The vertical shear vector is related to both speed and directional shear according to the identity $(\partial \mathbf{V}/\partial z)^2 = (\partial V/\partial z)^2 + V^2(\partial\alpha/\partial z)^2$ for α in radians. Therefore the Richardson number $g\theta^{-1}(\partial\theta/\partial z)/(\partial \mathbf{V}/\partial z)^2$ tends to be small when $V(\partial\alpha/\partial z)$ is large. But if $V(\partial\alpha/\partial z)$ is small as a result of a negligible direction change, a large vertical speed change can still produce a large vector shear and a small Ri number; i.e., a small Ri number does not imply that pronounced turning of the wind with height exists. As mentioned earlier, moderate or severe turbulence tends to occur at the boundaries of upper fronts or at the tropopause. These boundaries are characterized by significant changes in lapse rate, i.e., by large magnitudes of $\partial^2 T/\partial z^2$. In the present investigation we have combined terms for wind speed, wind direction change, and lapse rate change in a convenient way as the product $V(\partial\alpha/\partial z)(\partial^2 T/\partial z^2)$, and have tentatively called it the "turbulence index." The relationships between the turbulence index and Richardson's number under typical

conditions are illustrated in figure 1. In section 4 it will be shown that the Ri number and turbulence index computed from upper-air data are of approximately equivalent skill in identifying turbulent regions.

3. METEOROLOGICAL CONDITIONS ASSOCIATED WITH A CASE OF MODERATE TURBULENCE

A clear example of moderate turbulence of type A and the associated meteorological conditions is shown in figures 2–5. The observations were made by the Air Force Cambridge Research Laboratories' B-47 aircraft in a flight from Los Angeles (at right) to Bryce Canyon, Utah (at left) perpendicular to a strong jet stream [5]. The aircraft path and turbulent gust intensity are shown in figure 2a. The turbulent area occupied only a small portion of the jet stream, as is typical of such cases. The thermal field shows essentially barotropic conditions over most of the cross section and a very pronounced upper (or "jet") front in the lower left corner. Moderate turbulence lay within and along the lower boundary of this stable layer, and light turbulence extended upward into the jet core. The Doppler wind measurements of the aircraft (fig. 3) show that the jet core lay immediately above the stable layer. Strong horizontal and vertical wind shear were associated with this layer. The outstanding feature of the wind field in relation to turbulence was the pronounced change of wind direction in the turbulent region. The Richardson number (fig. 4a) had minimum values along the upper and lower boundaries of the stable layer. The observed turbulence corresponds well to the latter region. The quantity $V(\partial\alpha/\partial z)$ had maximum magnitudes in the turbulent region, and moderately large values also existed in non-turbulent regions south of the jet core. (These latter regions do not belong to type A because the horizontal shear is not cyclonic and there is no vertical change of lapse rate.) The quantity $\partial^2 T/\partial z^2$ (fig. 5a) had largest magnitudes at the frontal boundaries and tropopause. The turbulence index shown in figure 5b had a rather clear relationship to the turbulent region. Similar associations of turbulence and these particular meteorological quantities are apparent in other case studies [4, 13]. In the present case, turbulence near the jet core (type B) was not well developed.

4. TURBULENCE ANALYSIS FOR MARCH 12–24, 1962

Questions relating to the utility of upper-air data in identifying turbulent regions, and concerning objective methods of utilizing rawinsonde data for this purpose are obviously of operational importance. To obtain partial answers to these questions, we have compared values of several meteorological quantities to turbulence reports made by airline and military pilots over the United States. These data are from a special reporting period in March 1962 [1] and were made available to us by Colson

in the form of plotted maps. One could not investigate these problems without data of this type.

It is well known that meteorological factors related to turbulence often occur concurrently (for example, jet streams, horizontal and vertical shear, directional changes, small Ri numbers, and changes in lapse rate may all be related to a particular turbulent region). Consequently, the turbulence analyst could formulate a number of criteria based on different combinations of factors. Ideally, we wish to obtain criteria using the minimum number of factors that correctly identify turbulent regions. At the same time, it is desirable to minimize the volume of atmosphere identified as turbulent, i.e., to pinpoint the analysis.

From previous experience, we expected the Ri number and turbulence index to be useful quantities that could be calculated easily from upper-air data. These quantities (and their component terms such as wind shear and lapse rate) were calculated by electronic computer from card deck 645, obtained from the National Weather Records Center. However, instead of computing the term $V(\partial\alpha/\partial z)$ in the turbulence index, we computed the more familiar quantity $(fT/g)(V^2\partial\alpha/\partial z)$, which has the units $^\circ\text{K. sec.}^{-1}$. By use of the thermal wind relation, this latter quantity is a form of the term "thermal advection" that has been associated with turbulence by Keitz [7] and Schwerdtfeger and Radok [14]. Although the patterns of $V(\partial\alpha/\partial z)$ and thermal advection are similar, it appears that the term V^2 in thermal advection overemphasized high wind speeds. Therefore, the appropriate formulation of the turbulence index is $|V(\partial\alpha/\partial z)(\partial^2 T/\partial z^2)|$.

In the range from 500 to 200 mb., computations were made over 50-mb. intervals, except for $\partial^2 T/\partial z^2$, which was computed over 100-mb. intervals. Because from 200 to 100 mb. data are twice as frequent, the corresponding computational intervals were 25 and 50 mb. The computed quantities were printed at station locations on the same base map for which the turbulence observations were plotted, thus permitting immediate comparison of the computer output and turbulence reports. An example of a map of the magnitude of wind shear $|\partial V/\partial z|$ is shown in figure 6 for March 15, 1962. Four numbers in a row are printed at each rawinsonde station, applying to four 50-mb. intervals (500–450, 450–400, etc.). Dashes signify missing data. The shears have been scaled to arbitrary units so that the majority lie between 0 and 9, with 9 as an upper limit (to avoid printing two digits for a layer). The present units are $2 \times 10^{-3} \text{ sec.}^{-1}$ so that the printed number 4, for example, means a shear of $8 \times 10^{-3} \text{ sec.}^{-1}$ (4.75 kt./1000 ft.). Isotachs and jet streams at the 400-mb. level have also been indicated in figure 6. Most, but not all, of the layers of large shear are associated with high wind speeds. The letters T and N signify the presence or absence, respectively, of moderate or severe turbulence as shown by Colson's data. The turbulence reports shown are for altitudes between 18,000 and 29,000

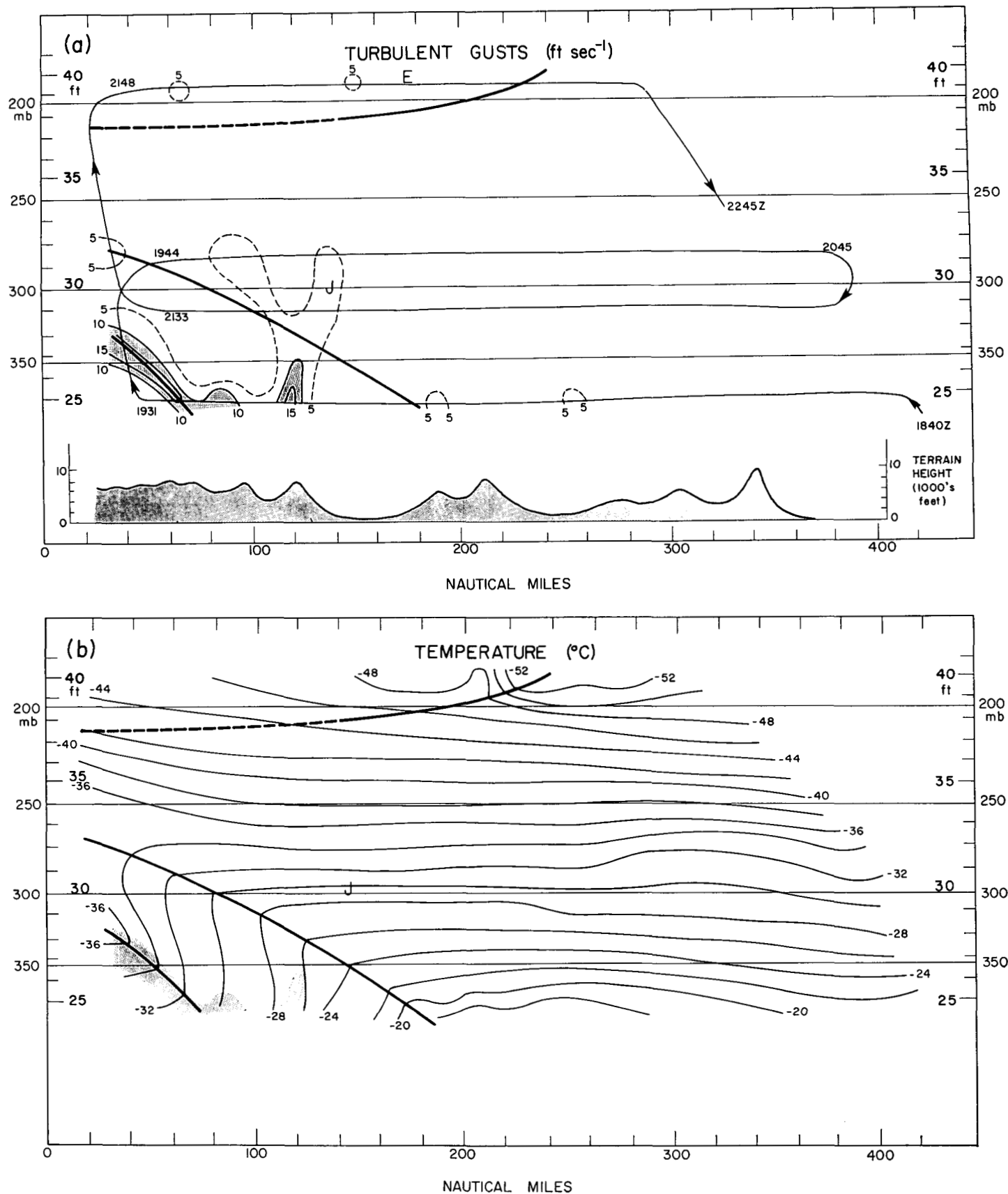


FIGURE 2.—Vertical cross section perpendicular to a jet stream, Flight E-36, February 25, 1958. Low pressure to left. The section extends from Bryce Canyon, Utah (at left end) to Los Angeles (at right end). Regions of moderate turbulence are shaded. (a). Aircraft path, time (GMT), and turbulent gust intensity. (b). Temperature measured by aircraft. Heavy solid lines are frontal boundaries or the tropopause.

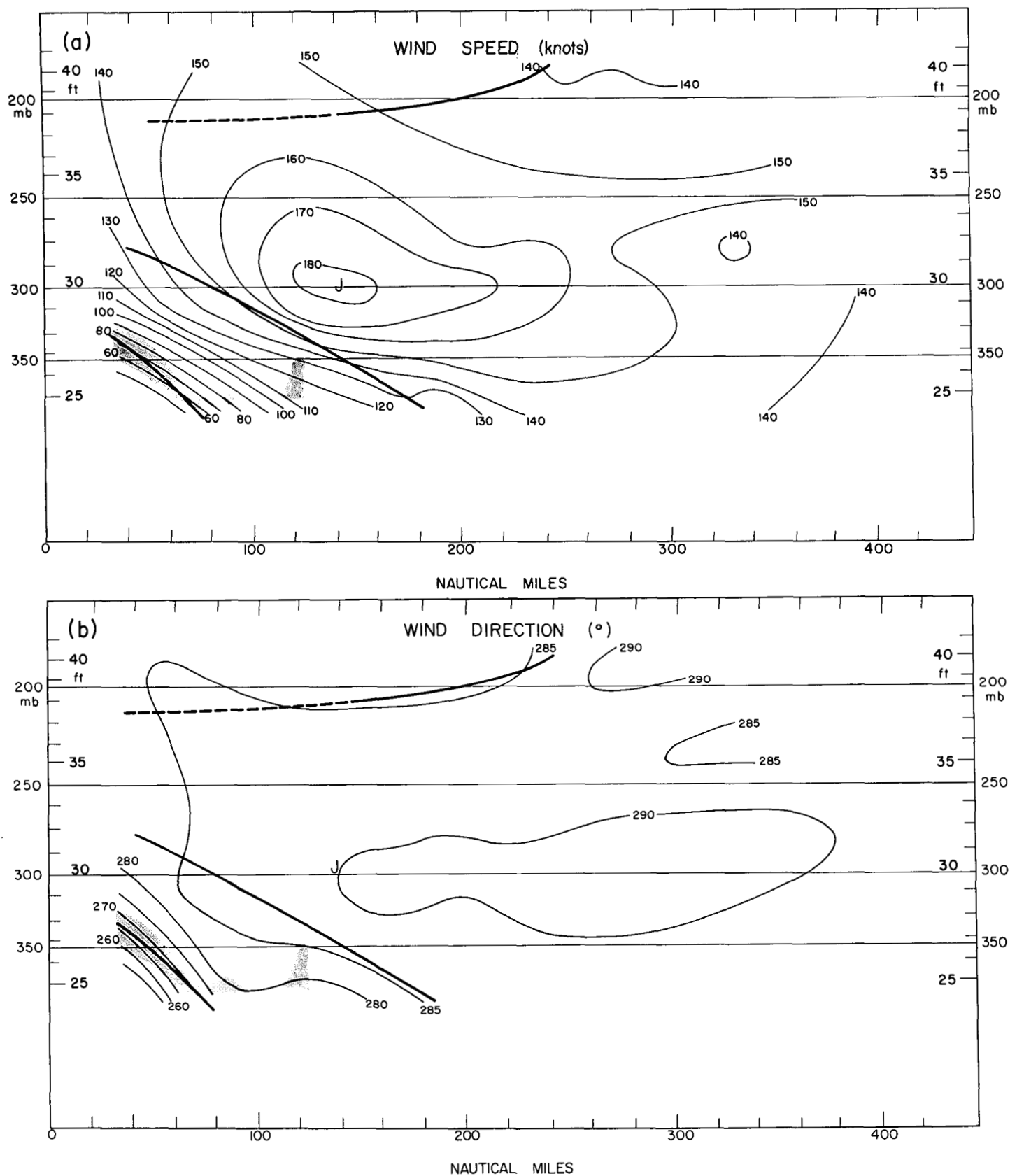


FIGURE 3.—Vertical cross section, Flight E-36. Regions of moderate turbulence shaded. (a). Wind speed measured by aircraft. J denotes the jet stream core. (b). Wind direction.

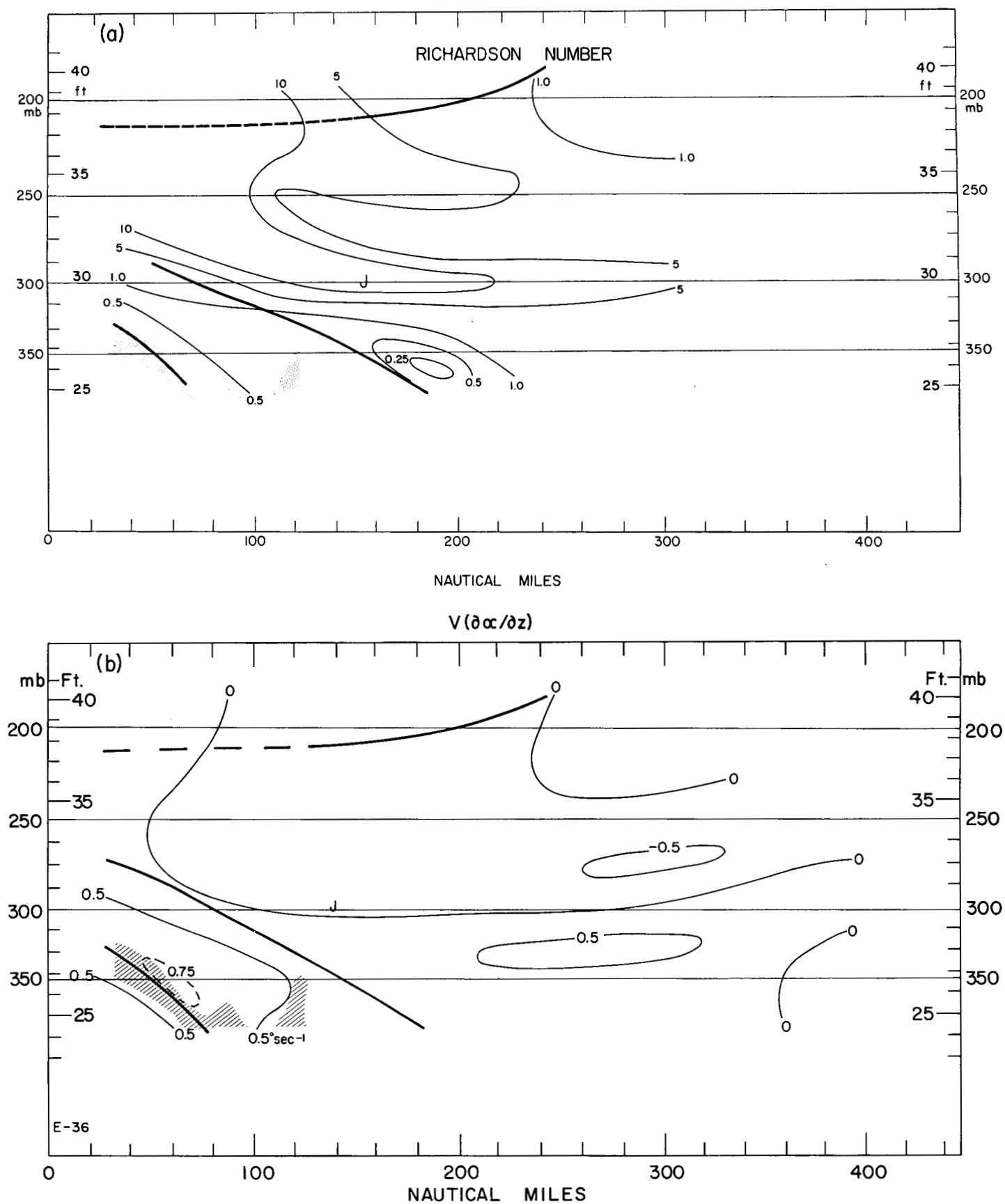


FIGURE 4.—Vertical cross section, Flight E-36. Regions of moderate turbulence shaded. (a). Richardson's number. (b). Product of wind speed (V , in m. sec.⁻¹) and vertical change of wind direction ($\partial\alpha/\partial z$, in deg. m.⁻¹).

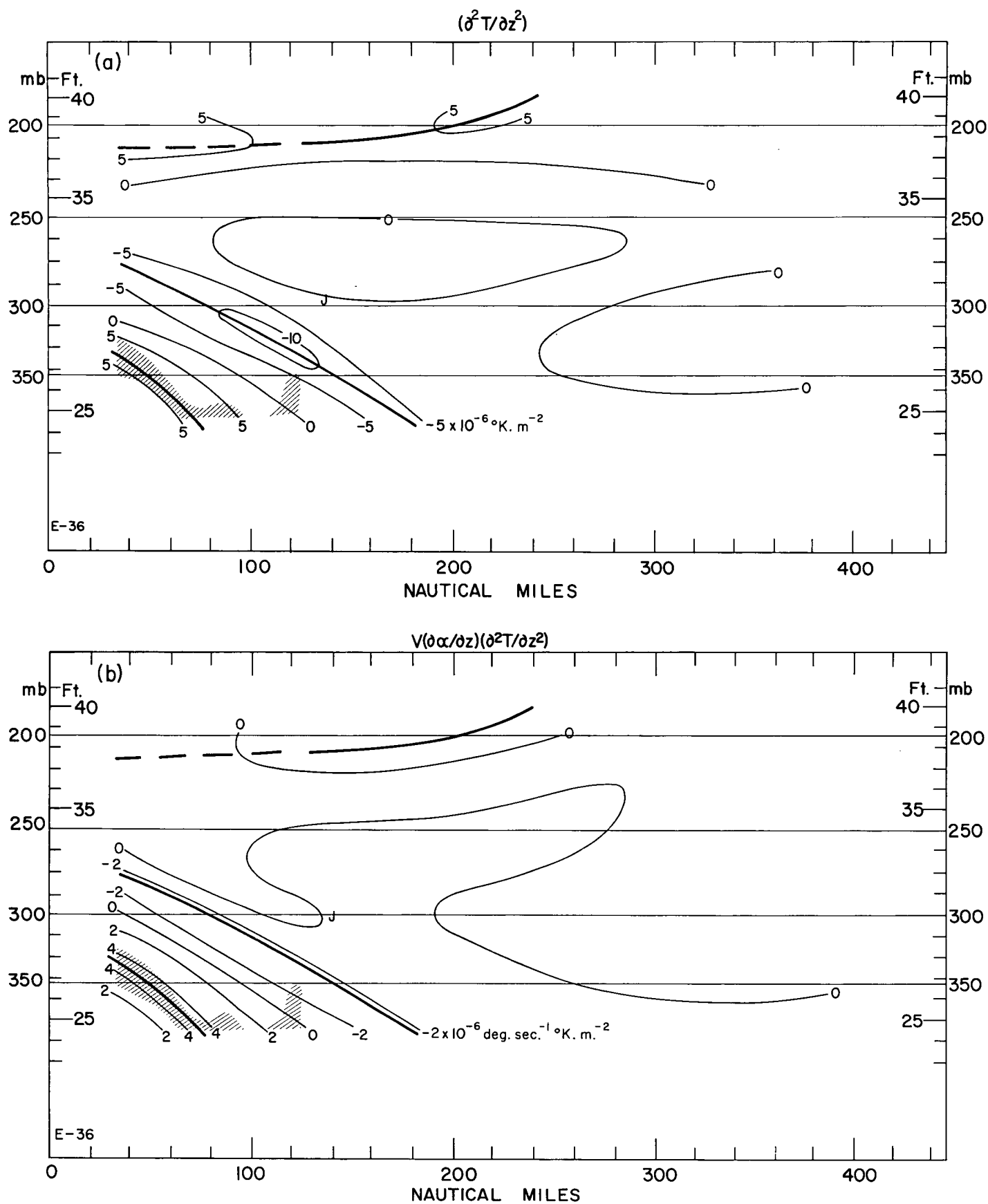


FIGURE 5.—Vertical cross section, Flight E-36. Regions of moderate turbulence are shaded. (a). Change of lapse rate $(\partial^2 T / \partial z^2)$, in $^{\circ}\text{K. m.}^{-2}$. (b). The product $V(\partial \alpha / \partial z)(\partial^2 T / \partial z^2)$ in $\text{deg. sec.}^{-1} ^{\circ}\text{K. m.}^{-2}$, called the turbulence index.

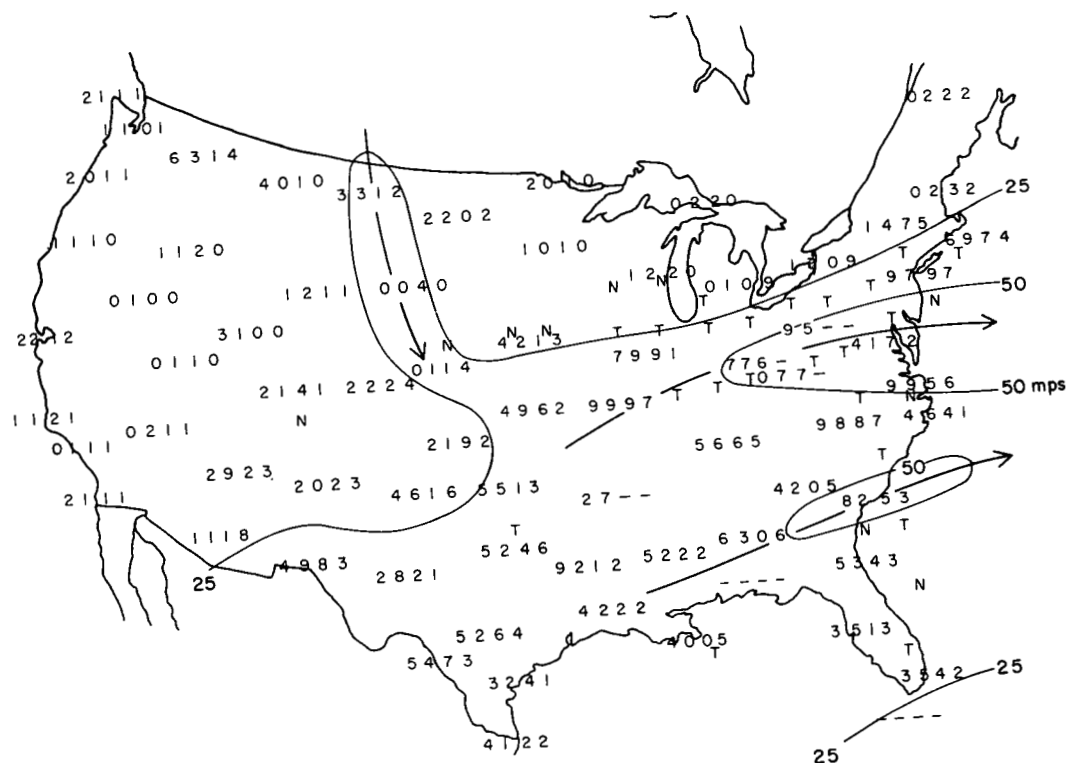


FIGURE 6.—Vertical wind shear over the United States, 0000 GMT, March 15, 1962. Four numbers at each station pertain to intervals 500–450 mb., 450–400 mb., 400–350 mb., and 350–300 mb. (from left to right). Shear is in the units $2 \times 10^{-3} \text{ sec}^{-1}$. Solid lines mark jet streams and principal isotachs. T denotes moderate or severe turbulence reported by aircraft; N denotes no moderate or severe turbulence reported.

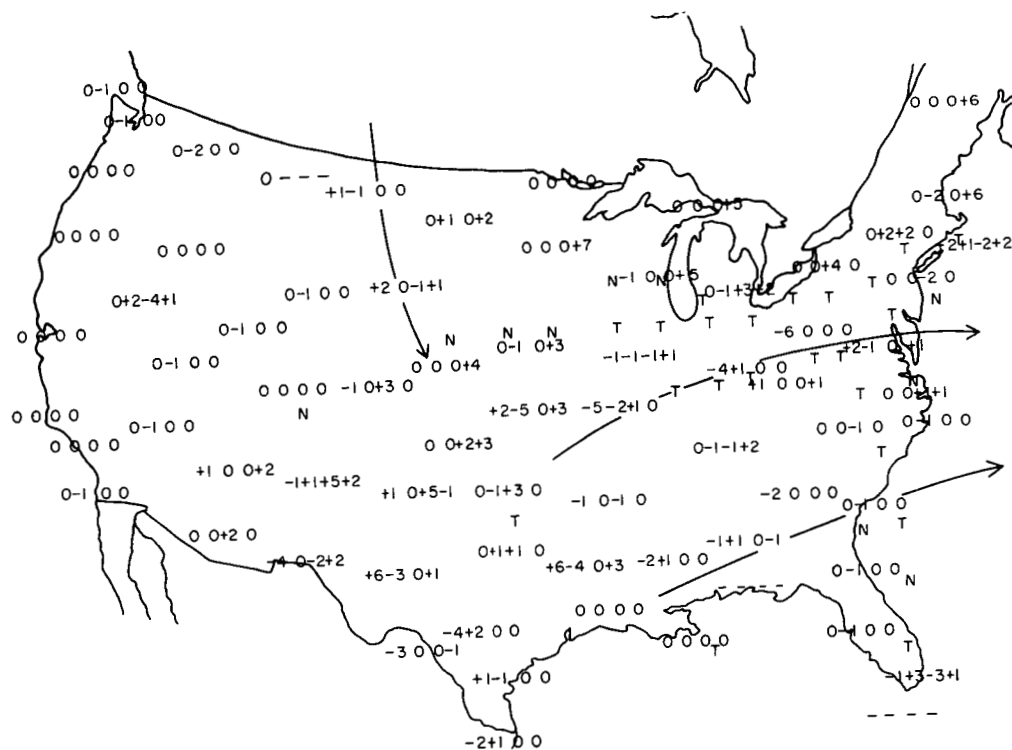


FIGURE 7.—The quantity $\partial^2 T / \partial z^2$ over the United States, 0000 GMT, March 15, 1962. Units of $10^{-6} \text{ } ^\circ\text{K. m}^{-2}$ for intervals 500–400 mb., 450–350 mb., 400–300 mb., and 350–250 mb. T denotes moderate or severe turbulence reported by aircraft; N denotes no moderate or severe turbulence reported.

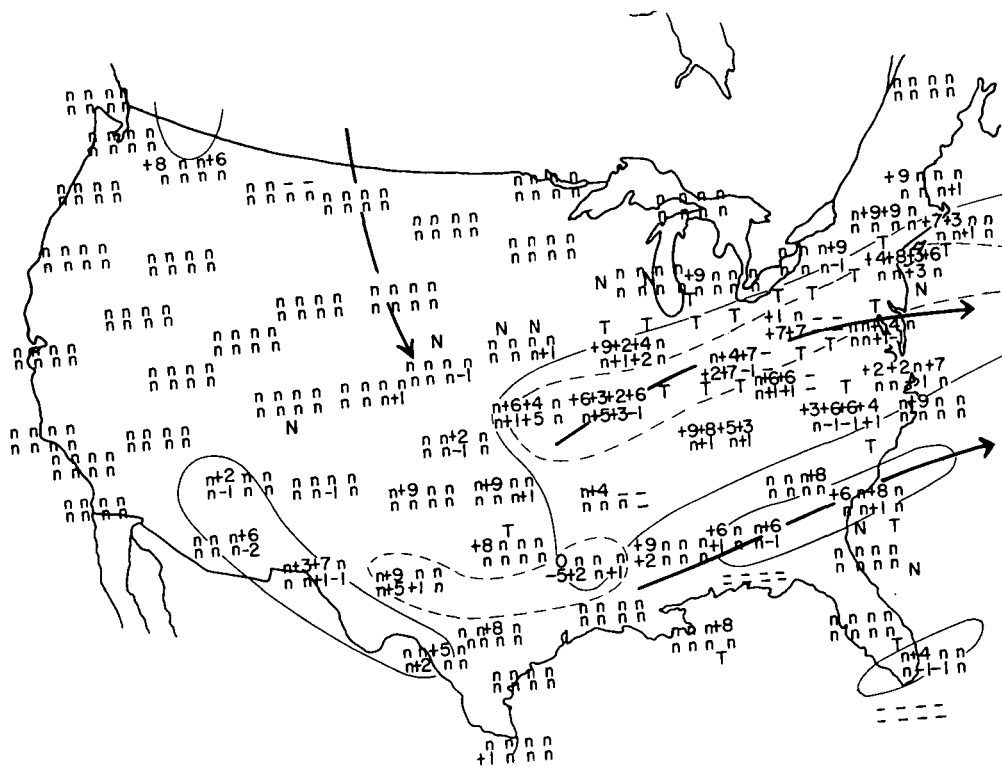


FIGURE 8.—Values of Richardson's number (upper row at each station) and turbulence index (lower row) for 0000 GMT, March 15, 1962 for intervals 500–450 mb., 450–400 mb., 400–350 mb., and 350–300 mb. n is used for $Ri \geq 1.0$ and for turbulence index < 1.0 . T denotes moderate or severe turbulence reported by aircraft; N denotes no moderate or severe turbulence reported. Heavy solid lines mark jet streams. Light solid lines enclose regions in which a value of $Ri \leq 0.6$ was computed; dashed lines enclose regions in which the turbulence index had a value ≥ 3 .

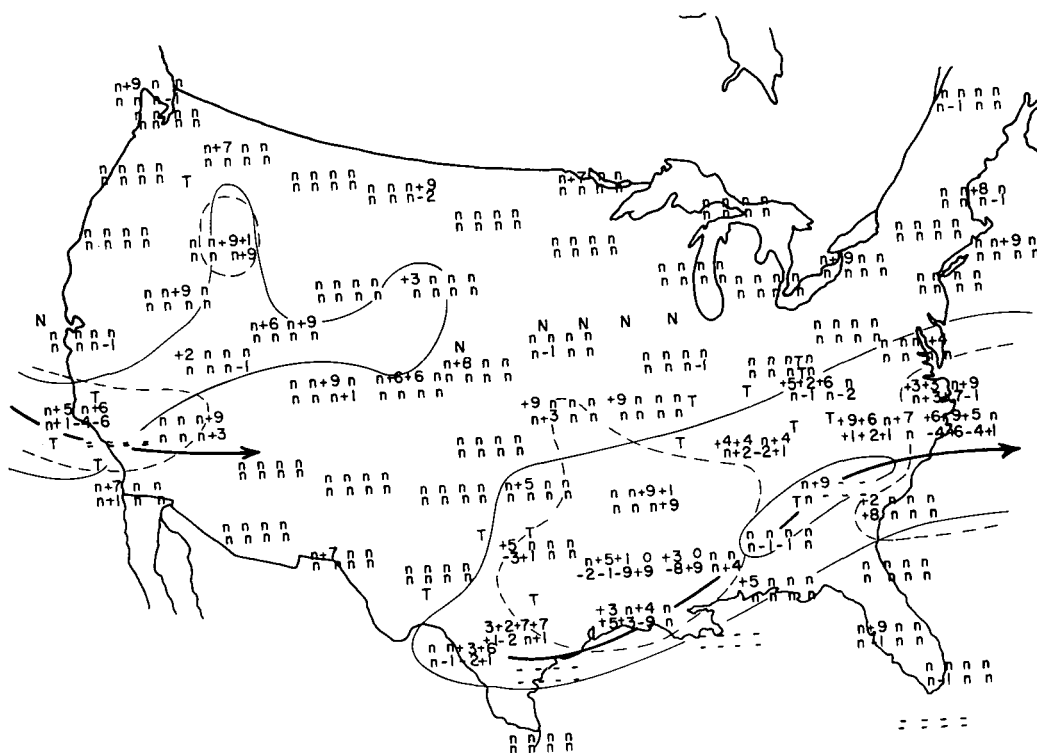


FIGURE 9.—Values of Richardson's number (upper row at each station) and turbulence index (lower row) for 0000 GMT, March 23, 1962 for intervals 500–450 mb., 450–400 mb., 400–350 mb., and 350–300 mb. n is used for $Ri \geq 1.0$ and for turbulence index ≤ 1.0 . See figure 8 for explanation.

ft. and fall within a 6-hour period centered at the map time. In order to make a reasonably clear differentiation between turbulent and smooth conditions, it was decided arbitrarily to apply the letter N only if three or more flights traversed a given region without encountering moderate or severe turbulence. One or more flights reporting moderate or severe turbulence is assigned the letter T, regardless of nonturbulent flights in the same area.

Computed values of the quantity $\partial^2 T / \partial z^2$ (units $10^{-6} \text{ } ^\circ\text{K. m.}^{-2}$) are shown in figure 7 for 100-mb. layers (500–400 mb., etc.). Recalling that large positive values indicate the base of stable layers and negative values their upper surfaces, we can quickly scan the numbers to locate such boundaries. For example, at Midland, Tex., the sequence +6 –3 0 +1 indicates the base and top of an inversion in the first two layers followed by little change in lapse rate in the two higher layers. A well-developed tropopause normally appears (in higher layers) as a large positive value of $\partial^2 T / \partial z^2$.

The computed values of Ri number (top row at each station) and turbulence index (bottom row) are shown in figure 8. It is interesting that the turbulence reports (T's) occupy a large continuous area south of the Great Lakes, extending from Illinois to New York. This turbulent region persisted for 18 hr. During this period, moderate turbulence was reported by approximately 50 percent of the flights made in each sector labeled by a T. In figure 8, small values of Ri and large magnitudes of the turbulence index indicate turbulence. Negative values of Ri (which have been noticed on a few occasions) imply superadiabatic lapse rates. Positive and negative signs have been retained in this presentation of the turbulence index; however, no utility has been found for the sign so that only the magnitude is of interest. Taking Peoria, Ill. (just south of Lake Michigan) as an example, in the upper row the sequence +9 +2 +4 n signifies Ri numbers of 0.9, 0.2, 0.4, and >1.0 (no turbulence expected). In the lower row, the sequence n +1 +2 n signifies values of the turbulence index less than 1 (no turbulence expected), +1, +2, and no turbulence. To aid in the inspection of figure 8, solid lines are used to enclose regions in which a value of $Ri \leq 0.6$ was computed, while dashed lines enclose regions in which the turbulence index had a value ≥ 3 .

At stations in the western United States, both the Ri number and turbulence index generally indicate that no turbulence should be expected. East of the Mississippi River, many layers are found having small values of Ri or large values of the turbulence index. The patterns of the two criteria match the turbulence reports quite well; however, in individual layers, the two criteria do not necessarily agree.

The quantities mentioned above were computed and printed for 0000 and 1200 GMT during the period March 12–24, 1962. In the interest of brevity, only one additional chart of this series will be mentioned. On March 23 (fig. 9), a very strong jet in advance of a

trough lay over the southeastern United States. Both turbulence criteria indicate unusual values along the jet; however, no aircraft data were available in this region. Farther away from the jet, the aircraft reports compared favorably with the computations.

In summary, the computed values of the Ri number and turbulence index were in qualitative agreement with turbulent intensity reported by aircraft during the period March 12–24, 1962. In regions where aircraft reports were not obtained the criteria indicate turbulent regions that are related to troughs, ridges, and jet streams as one would expect.

5. STATISTICAL ANALYSIS OF TURBULENCE CRITERIA

The series of maps of Ri number, turbulence index, and turbulence reports for the period March 12–24, 1962 were used to prepare tables showing the accuracy of these two criteria in turbulence analysis. The plotted T's (turbulent flights) and N's (no turbulence) were compared with the Ri number and turbulence index at the nearest station in the region. For the first test, $Ri < 0.6$ and turbulence index ≥ 3.0 (in arbitrary units) were chosen as denoting turbulence. For the criteria within these ranges, a T constitutes a correct identification and an N constitutes an error. Conversely, for $Ri \geq 0.6$ and turbulence index < 3.0 , an N is a correct identification, and a T is an error. The criteria were

TABLE 1.—Accuracy of analysis of moderate or severe turbulence using Ri number and turbulence index (TI) within specified ranges. The No-Skill tables show the numbers to be expected by chance. The skill score is shown by S. VOL is the percentage of volume having the specified conditions of Ri or turbulence index.

I $Ri \text{ NO.} < 0.6$

OBSERVED	ANALYZED		
	NON-T	TURB	
NON-T	191	16	207
TURB	86	34	120
	277	50	327

NO SKILL

175	32	$\chi^2 = 26.2$
102	18	$S = 0.24$
		$VOL = 4\%$

II $Ti \text{ NO.} \geq 3.0$

OBSERVED	ANALYZED		
	NON-T	TURB	
NON-T	186	21	207
TURB	80	40	120
	266	61	327

NO SKILL

168	39	$\chi^2 = 28.3$
98	22	$S = 0.26$
		$VOL = 3\%$

III $Ri \text{ NO.} < 0.6 \text{ AND/OR } Ti \text{ NO.} \geq 3.0$

OBSERVED	ANALYZED		
	NON-T	TURB	
NON-T	175	32	207
TURB	63	57	120
	238	89	327

NO SKILL

151	56	$\chi^2 = 38.2$
87	33	$S = 0.34$
		$VOL = 6\%$

tested separately, and also on an alternative basis, as shown in table 1.

Of the 327 cases that were available, 120 were turbulent and 207 were non-turbulent. Using $Ri < 0.6$ as a criterion, we correctly identify 34 (or 28 percent) of the turbulent cases and 191 (or 92 percent) of the non-turbulent cases. The numbers to be expected by chance are shown in the no-skill table. The value of χ^2 for the difference between the actual and no-skill tables is 26.2 which is significant at the 1 percent probability level. The skill score, S , (Panofsky and Brier [10]) is 0.24, where 0 denotes no skill and 1.0 denotes perfect agreement between observations and analysis. The number of intervals found to have values of $Ri < 0.6$ was 4 percent of the total. The optimum analysis will maximize the number of correct identifications of turbulence while minimizing the portion of atmosphere involved.

Using the turbulence index ≥ 3.0 as a criterion (Part II of table 1) gives 40 out of 120 (or 33 percent) correct identifications of turbulence and 186 out of 207 (or 90 percent) of the non-turbulent cases. Values of χ^2 , S , and percentage of volume are similar to Part I of table 1.

Since the Ri number and turbulence index represent somewhat different flow characteristics, it is natural to consider the success that can be obtained by use of $Ri < 0.6$ and/or turbulence index ≥ 3.0 as a turbulence criterion. From Part III of table 1, we see that the number of correct identifications of turbulence is increased to 57 out of 120 (48 percent), while 175 out of 207 (85

percent) of the non-turbulent areas are identified. Thus, an increase in the first percentage is accompanied by a decrease in the second. Stated differently, a larger number of hits in specifying turbulence is obtained at the cost of incorrectly identifying more non-turbulent areas as being turbulent. Since there is some overlap in the regions specified by the two criteria, the volume in Part III is slightly less than the sum of the two separate volumes.

The accuracy in identifying turbulent regions shown in table 1 is far less than we desire, even though positive skill has been demonstrated. If the criteria are relaxed, more correct identifications of turbulence are made, more errors are made in specifying non-turbulent regions as turbulent, and volumes of atmosphere involved are increased (table 2). Some of these errors may be due to a lack of flights at the altitude and place identified as turbulent, i.e., turbulence that existed may not have been detected by aircraft in the region. Skill scores and values of χ^2 in table 2 are similar to those of the previous table; however, the results of table 2 may be of greater operational value since 85 out of 120 (71 percent) of the turbulent cases are correctly identified.

These encouraging results may be due in part to the exclusion of light turbulence (which frequently occurs in small patches not clearly related to identifiable phenomena) from the turbulent (T) category. At any rate, positive skill in analyzing moderate or severe turbulence has been demonstrated using two quantities calculated directly from standard upper-air observations. No plotting or analysis precedes the computation. A quantity (the turbulence index) has been found that has accuracy equivalent to Richardson's number as a turbulence criterion, judged on the basis of a reasonably large sample of data. Combined use of the Ri number and turbulence index gives better analyses than sole use of either. These results have been obtained in spite of discrepancies which are due to the subjectivity of turbulence reports and to upper air data not perfectly coincident in time and space with the turbulence observations.

6. CONCLUDING REMARKS

Further studies of turbulence criteria should be directed to increasing the percentage of correct identifications of turbulent areas, and to reducing the volume identified as turbulent toward the 5-percent value that is realistic. It is our impression that improvement could be accomplished, in part, by de-emphasizing the high-pressure side of jet streams (where turbulence is sometimes absent even when Richardson's number is small and the turbulence index is large). Since vorticity has been shown to be related to turbulence [3], and because vorticity is large to the north of jets and small to the south, this quantity might be included in turbulence criteria. Or instead of vorticity ($\partial v / \partial x - \partial u / \partial y$), the product $(\partial v / \partial x)(\partial u / \partial y)$ might be used. In a typical upper-air pattern, this term will have largest magnitudes near troughs or ridges on the

TABLE 2.—Accuracy of analysis of moderate or severe turbulence using larger values of Ri and smaller values of the turbulence index than in table 1.

I Ri NO. < 1.0

ANALYZED				NO SKILL			
OBSERVED		NON-T	TURB				
	NON-T	172	35	207	145	62	$\chi^2 = 45.7$
	TURB	57	63	120	84	36	$S = 0.37$
		229	98	327			$VOL = 8\%$

II Ti NO. ≥ 2.0

ANALYZED				NO SKILL			
OBSERVED		NON-T	TURB				
	NON-T	164	43	207	146	61	$\chi^2 = 20.6$
	TURB	67	53	120	85	35	$S = 0.25$
		231	96	327			$VOL = 6\%$

III Ri NO. < 1.0 AND Ti NO. ≥ 2.0

ANALYZED				NO SKILL			
OBSERVED		NON-T	TURB				
	NON-T	141	66	207	111	96	$\chi^2 = 47.7$
	TURB	35	85	120	65	55	$S = 0.37$
		176	151	327			$VOL = 13\%$

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low-pressure side of jet streams, i.e., in regions that are often turbulent. For the March 15 data, the Scorer number $g\theta^{-1}(\partial\theta/\partial z)[V(\partial^2V/\partial z^2)]^{-1}$ was calculated. Although inaccuracies in wind data make the calculated second derivatives somewhat unreliable, this number appeared to identify some turbulent regions not indicated by Richardson's number or the turbulence index. Therefore, the Scorer number may have practical application. If a convenient means were found for calculating vertical motion in the upper troposphere on a sub-synoptic scale, this quantity might also be useful in turbulence analysis. This enumeration is not intended to be exhaustive, but rather to indicate types of parameters that have potential use in improving turbulence analysis.

In evaluating the *Ri* number and turbulence index (or any other criteria) for clear air turbulence, it would be desirable to have a record of the intensity of turbulence in each layer at each station, obtained by a sensor flown with the radiosonde instrument. In that case, one could determine to what extent turbulence intensity is related quantitatively to the criteria. Moreover, the amount of useful data would be tremendously enlarged, and operational analysis would be greatly facilitated. As matters stand, comparisons of turbulence reports and meteorological quantities are confined mainly to major air routes.

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{Received August 24, 1964}